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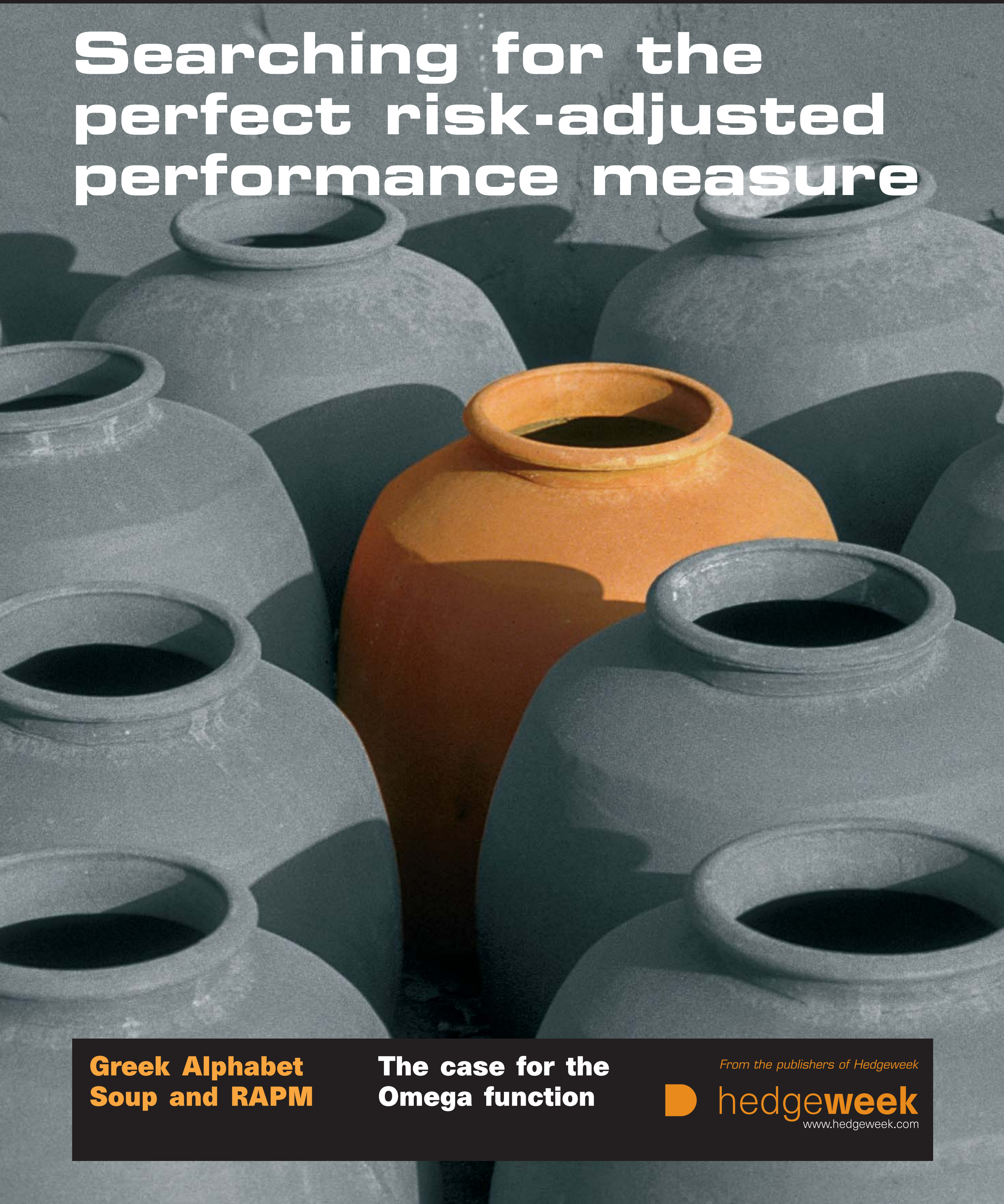
Searching for the perfect risk-adjusted performance measure



Greek Alphabet Soup and RAPM

The case for the Omega function

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'Sharper' Risk Adjusted Performance Measures (RAPMs): From Omega to AIRAP

by Milind Sharma

Two measures that address weaknesses of the Sharpe ratio are the Omega measure and Alternative Investments Risk Adjusted Performance metric

The problem: traditional RAPMs can mislead

The most widely used traditional risk-adjusted performance measure (RAPM) is the Sharpe ratio, coined by its namesake and Nobel laureate William Sharpe in 1966. It has many desirable properties such as proportionality to the t-statistic (for returns in excess of zero) and the centrality of Sharpe-squared to optimal portfolio allocation. However, it is leverage invariant; it does not account for correlations; nor can it handle iceberg risks lurking in the higher moments. Worse yet, it can be 'gamed' by truncating the right tail of the returns distribution at the expense of a fat left tail (the periodic crashes).

A team of Yale professors has derived the optimal strategy to manipulate the Sharpe ratio, which comprises of shorting out of the money puts and calls in a specific ratio. They remark that, "the 'peso problem' may be ubiquitous in any investment management industry that rewards high Sharpe ratio managers." It is now widely recognized that new RAPMs are required to deal with the complexities of the HF paradigm. An in-depth survey of emerging state of the art tools can be found in Barry

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Schachter's 2004 compilation, *Intelligent Hedge Fund Investing*. The author's research [Sharma (2004b)] has confirmed that high Sharpe ratios in hedge funds often represent a trade-off for higher moment risk.

One line of thought is to salvage the Sharpe ratio's relevance while retaining the familiar form by replacing standard deviation in the denominator with an enhanced risk measure such as Modified VaR or AIRAP. In the parametric VaR case, assuming normality of returns, one obtains at the 99% confidence level:

$$VaR_{NORMAL} = \mu - 2.32\sigma$$

The Cornish-Fisher expansion shown below can be used to modify VaR in order to include the impact of the skewness and kurtosis:

$$VaR_{CORNISH-FISHER} = \mu - \Omega(\alpha)\sigma$$

$$\Omega(\alpha) = z(\alpha) + \frac{(z(\alpha)^2 - 1)S}{6} + \frac{(z(\alpha)^3 - 3z(\alpha))K}{24} - \frac{(2z(\alpha)^3 - 5z(\alpha))S^2}{36}$$

Where (1-α) is the confidence level, $z(\alpha)$ the critical value under normality, S is skewness, and K is excess kurtosis. Thus, the alternative formulations are:

$$\text{Modified Sharpe Ratio} = {(\mu - r_f)}\Big/{VaR_{CORNISH-FISHER}}$$

or

$$\text{Modified Sharpe Ratio} = {(Mean\_excess\_return)}\Big/{AIRAP\_RP(4)}$$

where AIRAP RP(4) is the AIRAP based Risk Premium for the default value of CRRA = 4.

However, these ratios do inherit some of the limitations of the Sharpe ratio in addition to the fact that a four moment approximation does not include all higher moments and comes with its own convergence issues.

New alternative RAPMs for hedge funds: Omega and AIRAP

We now summarize two key alternatives for a new HF risk-adjusted performance measure:

i) Gain-Loss ratios such as that originally due to Antonio Bernardo and Olivier Ledoit [Bernardo and Ledoit (2000)] or a generalized version called Omega proposed by Con Keating and William Shadwick [Shadwick and Keating (2002)];

ii) utility based measures such as AIRAP proposed by the author that explicitly factor in risk-aversion. [Sharma (2004a)]

Omega/ Gain-Loss Ratios: Omega takes the ratio of the expectations above and below a given threshold L as shown in the formula below:

$$\text{Omega(L)} = \left(\int_L^b (1-F(x))dx\right) \Big/ \left(\int_a^L F(x)dx\right)$$

The Omega function is calculated for all values of L in the interval [a,b]. Higher pointwise values of Omega are indicative of an investment with better upside to downside at those threshold points. The special case where the threshold L is zero corresponds to the ratio of gains to losses. Omega is mathematically equivalent to the returns distribution and hence incorporates all moments, although empirical results suggest that it is most sensitive to changes in mean and variance. Omega is monotonically decreasing from infinity to zero.

An intuitive restatement of Omega by Thomas Schneeweis and the CISDM faculty, [Kazemi et al] shows that it is essentially the ratio of a hypothetical European call and put on the underlying fund investment:

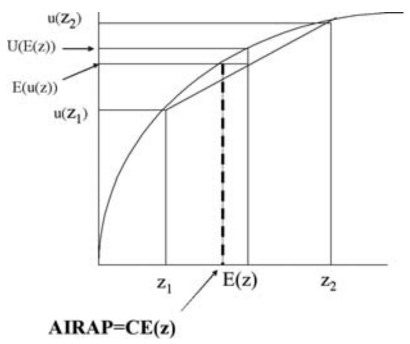
Omega(L) = C(L)/P(L)

Omega does not explicitly factor in any risk-aversion parameter nor is a default value of the threshold L prescribed (to facilitate comparisons). While the lack of a simple threshold or/ single point of comparison across funds may be a source of confusion, it is clear that the choice of L used in analysis should be consistent with some notion of investor risk-aversion. The main benefit is that it Omega captures all observed higher moments without making restrictive distributional assumptions. Finally, an empirical limitation is sample size. Stability of estimates requires at least 40 to 50 observations.

AIRAP (Alternative Investments Risk Adjusted Performance): AIRAP has been proposed by the author (Sharma) [Sharma (2004a)] as the certainty equivalent risk-adjusted return corresponding to a CRRA (constant relative risk-aversion) representation of investor preferences. Simply put it is the implied equivalent return that the risk-averse investor desires with certainty in exchange for the uncertain return from holding risky assets. It allows us to decompose return into the risk premium earned and the risk-adjusted (AIRAP) component, thus enabling an apple-s to-apple facilitating an equitable comparison of HF performance.

A graphical interpretation of this the certainty equivalent return is shown in figure x below. The vertical axis is utility and the horizontal axis is return (or wealth). The shape of the utility function (the curved line showing utility for any given level of return) is concave, which captures the economic idea of risk aversion. A combination of returns z1 and z2 has the expected (ie probability-weighted) value E(z). Risk aversion means the investor value this combination less highly (so it lies beneath the utility function

Figure 1: A concave utility function illustrating the idea of AIRAP or certainty equivalence



curve) of the certain outcome with the same value. The certainty equivalent of the probabilistic outcome is the lower value that generates the same utility for the investor, so the gap between CE(z) and E(z) is a kind of insurance premium.

For *TRi* = *i*th period total fund return, *c* = CRRA risk-aversion parameter, *i* = 1,...,N and N = number of periods, the general solution is reproduced below. A default value of c=4 for risk-aversion is recommended to facilitate comparisons. Further, substituting pi = probability of the ith return= 1/N, provides a closed form solution that has a straightforward spreadsheet implementation.

$$AIRAP = CE = \left[\sum_i p_i.(1+TR_i)^{(1-c)}\right]^{1/(1-c)} - 1, \quad when \quad c \neq 1 \& c \geq 0$$

$$and \ AIRAP = \left[\prod_i (1+TR_i)^{p_i}\right] - 1, \quad when \quad c = 1$$

AIRAP's key merits are that: it captures all observed higher moments, penalizes for volatility and leverage in proportion with risk aversion; it works even when mean returns are negative; it can be formulated as a modified Sharpe ratio; downside variance is penalized more; it is invariant to wealth level and the closed form solution is as simple to calculate as the Sharpe ratio in a spreadsheet. The unrealistic assumption of normality is avoided and the limitations of mean-variance world are circumnavigated. Finally, AIRAP is unique in providing insights into the optimal level of leverage consistent with a given HF strategy.

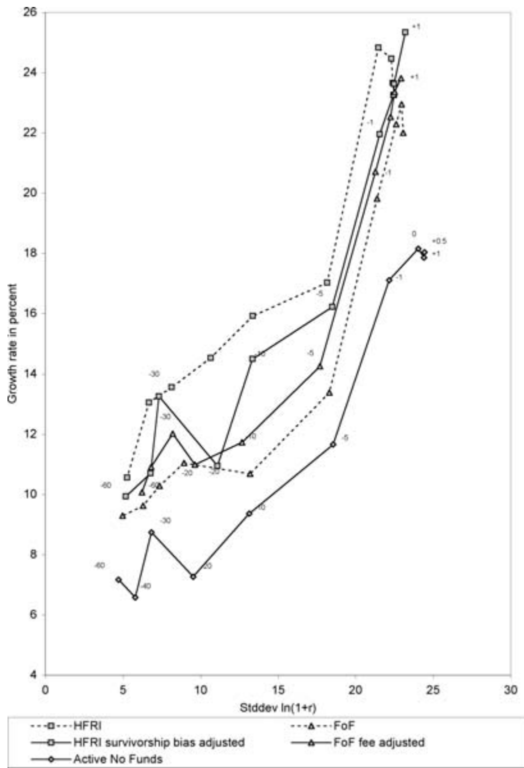
Using the new RAPMs: investing in HFs still stacks up

The question remains - is the investor better off investing in HFs net of the higher moment risks assumed? The answer is in the affirmative but is subject to further research. Bypassing the zigzag of discovery we hone in on We highlight recent results, which revisit the optimal asset allocation problem using new RAPMs relevant to the HF paradigm. Jean-François Bacmann and Sebastian Pache Pache [Bacmann and Pache (2004)] investigate optimization with Omega and show that resulting portfolios are less prone to overweighting negatively skewed and leptokurtic styles than under mean-variance optimization. Maximization of the Omega ratio corresponds to maximizing the ratio of expected gains and losses with respect to some threshold.

The proof of the pudding lies in the eating. Since investors ultimately eat returns, it is heartening to note that their study attributes the highest outsample returns to optimizing RAPMs incorporating higher moments - evidence that investors ought to care, if not for the theoretical underpinnings, then at least for their bottom --lines the sake of higher returns.

Maximizing AIRAP is tantamount to maximizing power utility. Operating in this framework, Bengt Pramborg and Niclas Hagelin (2004) have shown that even upon factoring in higher moment risks and survivorship corrections, it remains optimal to make significant allocations to hedge funds and FoHFs (figure 2 and table 1). In fact, they show that at times less risk averse investors may optimally choose to lever up and allocate all capital to HFs as proxied by the HFR composite index. Even the more risk averse investor would allocate as much to HFs as to equities (and significantly more than to bonds).

Figure 2



Source: Hagelin and Pramborg (2004)

Table 1: Differences in growth rates between portfolios with and without adjusted hedge fund indices

*The table displays the percentage differences in growth rates between portfolios for which investments in hedge funds are allowed and portfolios without hedge funds. Fund indices are adjusted for survivorship bias (HFRI), and for fees (FoF). Significance at the 10 percent level, at the 5 percent level, and at the 1 percent level is marked by *, **, and ***, respectively.*

	No Leverage Equity is SP500		Leverage Allowed Equity is SP500		Leverage Allowed Equity is MSCIW	
Portfolio Strategy	HFRI	FoF	HFRI	FoF	HFRI	FoF
EW	1.3***	0.5	1.3**	0.1	1.4***	1.0*
-60	2.0**	0.5	2.7**	2.8*	0	3.2**
-40	0.6	0.4	4.1***	4.3**	4.8***	3.4**
-30	1.8*	0.4	4.5**	3.2*	4.6***	2.8*
-20	1.8*	0.1	3.6**	3.7**	5.1**	1.3
-10	1.5	0.8	5.1**	2.3	8.0***	2.8
-5	3.1**	2.3*	4.5**	2.6	7.2***	2.6
-1	1.9	1.4	4.8**	3.5	4.9*	-0.1
0	2.1	2	5.1*	4.3	5.2	1
0.5	2.9*	2.5*	5.7*	5.4*	4.6	1.8
1	3.4**	2.5*	7.3**	5.7**	4.1	1.2

Source: Hagelin and Pramborg (2004)

Furthermore, they show that the incremental performance gains resulting from the inclusion of HFs to the traditional stock and bond mix can be both statistically significant and quite substantial. For the risk neutral investor the pickup in annualized geometric mean return ranges from 3.4% to 7.3% (when 2 times leverage is allowed), even after adjusting for survivorship bias and keeping volatility fixed.

Concluding thoughts

The rigorous study of HFs is still in its infancy. We provide some parting thoughts for the investors on measuring performance:

- Maintain the distinction between ex-post RAPM comparisons and their ex-ante relevance to out-sample performance.
- An assessment of investor risk aversion and loss threshold is critical to implementing paradigm RAPMs such as AIRAP and Omega. Be wary of traditional RAPM comparisons. Use of AIRAP or Omega would be prudent. HFs have many attractions and including them in asset allocation ought to be both desirable and optimal. ■